9. Illumination and Shading

Approaches for visual realism:

1. Remove hidden surfaces
2. Shade the visible surfaces and reproduce shadows
3. Reproduce surface properties:
   - texture
   - degree of transparency,
   - roughness, ... etc

9.1 How to shade a visible surface? depends on

- position
- orientation
- characteristics of the surfaces
- light sources
Light sources:

- Light-emitting sources -- light bulb, sun, ...
- Light-reflecting sources -- illuminated surfaces of other objects

Reflection Type:

- **diffuse reflection**: reflections scattered in all directions

- **specular reflection**: reflections point in a range of direction only
Ambient light (background light)
- results of multiple reflections from nearby objects
- can be considered to be of uniform intensity $I_a$ in all directions
- use a general level of brightness to model it to produce a uniform illumination
- reflected in all directions

Shade at a point of a surface

$$= \text{intensity of light reflection from that point of the surface}$$

$$= \text{diffuse reflection of ambient light}$$

$$+ \text{diffuse and specular reflection of one or more point light sources}$$

$$+ \text{diffuse and specular reflection of transparent effect}$$
How to estimate the intensities of these?

**Lambert’s (cosine) Law:**

\[
\text{Diffuse reflection} = I_p \cdot k_d \cdot \cos \theta
\]

- \(I_p\): intensity of light source
- \(k_d\): coefficient of reflection

\[
\begin{align*}
    k_d \rightarrow 0 & \quad \text{surface is highly absorptive} \\
    k_d \rightarrow 1 & \quad \text{surface is highly reflective}
\end{align*}
\]
Phong Model:

\[
\text{Specular reflection} = I_p \cdot W(\theta) \cdot \cos^n \alpha
\]

- **Specular reflection** models must produce the highest intensity in the direction of \( R \), with the intensity decreasing rapidly as the viewing angle \( \alpha \) increases.

**Guidelines:**

- Shining surface \((n \geq 200)\)
- Rough surface \((1 \leq n \leq 200)\)

For glass

- \( W(\theta) \to 1 \) if \( \theta \to 90^\circ \)
- \( W(\theta) \to 0 \) if \( \theta \to 0^\circ \)

Other materials have constant \( W(\theta) \)
Effect of Transparency:

Transparency effect = $I_{pt} \cdot k_{pt}$

$I_{pt}$: intensity arriving from behind

($(= I_a$ if no light source behind object)$

$k_{pt}$: transmission coefficient

depends on the material

Hence, shade at point $P$ (viewpoint at infinity)

$$I = I_a k_d + I_p \left[ k_d \cos \theta + W(\theta) \cos^n \alpha \right] + I_{pt} k_{pt}$$

or (viewpoint not at infinity)

$$I = I_a k_d + I_p \left[ k_d \cos \theta + W(\theta) \cos^n \alpha \right] / (r + r_0) + I_{pt} k_{pt}$$

$r$: distance from viewpoint to $P$; $r_0$: constant
How to compute $R$?

In the above formula, if $L$, $N$, $R$, and $V$ are normalized, we have

$$\cos \theta = L \cdot N$$

$$\cos \alpha = R \cdot V$$

But $R =$?

$$R = 2(N \cdot L)N - L$$
9.2 displaying Light Intensities

- Values of intensity calculated by an illumination model must be converted to one of the allowable intensity levels for the particular graphics system in use.

How should intensity levels be spaced?

- logarithmically, not linearly.

- Eye is sensitive to ratios of intensity levels rather than to their absolute values.

- Ratio of successive intensities should be constant to get equal perceived brightness.
For each component of the RGB model:

$I_0$: lowest attainable intensity (0.005 - 0.025)

$n + 1$: number of intensity levels with equal perceived brightness

We should have

$$\frac{I_1}{I_0} = \frac{I_2}{I_1} = \cdots = \frac{I_n}{I_{n-1}} = r$$

with

$$r = \left(\frac{1}{I_0}\right)^{1/n}$$

Example:

$I_0 = 1/8$, $n = 3$, $r = 2$, $I = 1/8, 1/4, 1/2, 1$

$I_0 = 1/8$, $n = 255$, $r = 1.0182$, $I = 0.0100, 0.0100, \cdots$
How to display a desired intensity $I$?

1. Determine the nearest $I_j$

   $$j = \text{ROUND} \left( \log_r \left( \frac{I}{I_0} \right) \right)$$

2. Calculate

   $$I_j = r^j I_0$$

3. Determine the pixel value

   $$V_j = \text{ROUND} \left( \frac{I_j}{K} \right)^{1/\gamma}$$

   (Gamma correction of intensity) Why?

4. If the raster display has no look-up table, then $V_j$ is placed in the appropriate pixel. Otherwise, $j$ is placed in the pixel and $V_j$ is placed in entry $j$ of the table.
Here is "Why?"

Relationship between intensity of light output (displayed intensity) and the number of electrons in the beam:

\[ I \propto N^\gamma, \quad 2.2 \leq \gamma \leq 2.5 \]

Relationship between the number of electrons in the beam and intensity value specified for the pixel (input voltage):

\[ N \propto V \]

Hence, to display a particular intensity value \( I \) for the pixel, the correct voltage value to produce this intensity is:

\[ V = \left( \frac{I}{K} \right)^{1/\gamma} \]

for some constant \( K \).
9.3 Halftone Approximation

How to expand the range of available intensities?

"Using the half toning technique to take advantage of the spatial integration phenomenon of human vision"

Spatial integration: view a small area from a sufficiently large viewing distance, the eye will integrate fine detail within the small area and record only the overall intensity of the area

halftoning: imprinting each small resolution unit with a circle of blank ink whose area is proportional to the blackness of the area in the original photo

How can the "halftoning" technique be used in computer graphics?

- Approximating the variable-area dots of halftone using multiple display pixels for one image pixel (trading spatial resolution for intensity resolution)
Example: $2 \times 2$ pixel area of a bi-level display

![Bi-level display examples](image)

(spatial resolution is cut by one-half)

Example: $2 \times 2$ pixel area of a 4-level display

![4-level display examples](image)

(Pattern should be designed to minimize contouring effects)
A Challenge: how to display $m \times m$ image with multiple levels of intensity on an $m \times m$ bi-level display? (i.e., approximating halftones without reducing resolution)

- using ordered dither technique to add dither noise over an entire picture

- a pixel $(x, y)$ to be intensified or not depends on the desired intensity $I(x, y)$ and on an $n \times n$ dither matrix $D^{(n)}$

Example:

$$n = 2, \quad \text{and} \quad D^{(2)} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

The display pixel $(x, y)$ is intensified if

$$I(x, y) > D^{(2)}_{i,j}$$

where

$$i = x \mod 2, \quad j = y \mod 2$$
Recurrence Relations for dither matrices:

\[ D^{(2n)} = (D^{(2n)}_{(l,k)})_{0 \leq l, k \leq 2n - 1} \]

1. If \((l, k) = 2 (i, j)\) for some \(0 \leq i, j \leq n - 1\) then

\[ D^{(2n)}_{(l,k)} = D^{(n)}_{(i,j)} \]

2. If \((l, k) = 2 (i, j) + (1, 1)\) for some \(0 \leq i, j \leq n - 1\) then

\[ D^{(2n)}_{(l,k)} = D^{(n)}_{(i,j)} + n^2 \]

3. If \((l, k) = 2 (i, j) + (0, 1)\) for some \(0 \leq i, j \leq n - 1\) then

\[ D^{(2n)}_{(l,k)} = D^{(n)}_{(i,j)} + 2n^2 \]

4. If \((l, k) = 2 (i, j) + (1, 0)\) for some \(0 \leq i, j \leq n - 1\) then

\[ D^{(2n)}_{(l,k)} = D^{(n)}_{(i,j)} + 3n^2 \]
Example:

\[ D^{(2)} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} \]

\[ D^{(4)} = \begin{bmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{bmatrix} \]