1. Evaluate each of the following:
   (a) \( \binom{10}{4} \)  
   (b) \( \binom{12}{7} \)  
   (c) \( \binom{14}{12} \)  
   (d) \( \binom{15}{10} \)

Sol.
   (a) \( \binom{10}{4} = \frac{10!}{4! \cdot 6!} = 210 \)
   (b) \( \binom{12}{7} = \frac{12!}{7! \cdot 5!} = 720 \)
   (c) \( \binom{14}{12} = \frac{14!}{12! \cdot 2!} = 91 \)
   (d) \( \binom{15}{10} = \frac{15!}{10! \cdot 5!} = 3003 \)

2. In how many ways can a gambler draw five cards from a standard deck and get
   (a) a flush (five cards of the same suit)? (b) four aces? (c) four of a kind? (d) three aces and two jacks?

Sol.
   (a) \( \binom{13}{5} \cdot 4 \)
   (b) 48
   (c) 13 \cdot 48
   (d) \( \binom{4}{3} \cdot \binom{4}{2} = 4 \cdot 6 = 24 \)

3. In how many ways can 12 different books be distributed among four children so that
   (a) each child gets three books? (b) the two oldest children get four books each and the two youngest get two books each?

Sol.
   (a) \( \binom{12}{3} \cdot \binom{9}{3} \cdot \binom{6}{3} \)
   (b) \( \binom{12}{4} \cdot \binom{8}{4} \cdot \binom{4}{2} \)

4. For any positive integer \( n \) determine
   (a) \( \sum_{i=0}^{n} \frac{1}{i! (n-i)!} \)
   (b) \( \sum_{i=0}^{n} \frac{(-1)^i}{i! (n-i)!} \)

Sol.
   (a) \( 2^n / n! \)
   (b) 0

5. In how many ways can 15 (identical) candy bars be distributed among five children so that
   the youngest gets only one or two of them?

Sol.
The youngest gets only one of them: $C(17,3)$
The youngest gets two of them: $C(16,3)$
Hence, the total is: $C(17,3) + C(16,3)$.

6. In how many ways can a teacher distribute eight chocolate donuts and seven jelly donuts among three student helpers if each helper wants at least one donut of each kind?

Sol. $C(7,2) \times C(6,2)$

7. Determine the number of integer solutions for
   
   $x_1 + x_2 + x_3 + x_4 + x_5 < 40$

   where

   (a) $x_i \geq 0$, $1 \leq i \leq 5$
   (b) $x_i \geq -3$, $1 \leq i \leq 5$

Sol.
   (a) $C(44,5)$
   (b) $C(59,5)$

8. In the following program segment, $i$, $j$, and $k$ and $counter$ are integer variables. Determine the value that the variable $counter$ will have after the segment is executed.

   ```
   counter := 10
   for i := 1 to 15 do
     for j := i to 15 do
       for k := j to 15 do
         counter := counter + 1
   ```

Sol.
   (a) Approach 1: $10 + \sum_{i=1}^{15} \frac{i^2 + i}{2} = 690$
   (b) Approach 2: Since $1 \leq i \leq j \leq k \leq 15$, we have $10 + C(15+3-1,15-1) = 10 + C(17,14) = 690$. 