1. [Simpson’s 1/3 rule]

In the derivation of the quadrature formula

\[ Q_n(f) = I(P_n,a,b) = a_0f_0 + \cdots + a_nf_n \]

if \( n = 2 \), the coefficients \( a_0, a_1 \) and \( a_2 \) would be \( \frac{b-a}{6} \), \( \frac{4(b-a)}{6} \) and \( \frac{(b-a)}{6} \), respectively. Why?

Sol. Because in this case (i.e., \( n = 2 \)),

\[ a_0 = \int_a^b l_0(x)dx = \int_a^b \frac{(x-a+b)(x-b)}{2} \frac{2}{(a+b)(a-b)} \]

\[ = \frac{2}{(a-b)^2} \int_a^b (x^2 - \frac{a+3b}{2}x + \frac{ab+b^2}{2})dx \]

\[ = \frac{2}{(a-b)^2} \left( \frac{x^3}{3} - \frac{a+3b}{4}x^2 + \frac{ab+b^2}{2}x \right) \bigg|_a^b \]

\[ = \frac{b-a}{6} \]

\( a_1 \) and \( a_2 \) can be computed similarly.

2. Give me all the reasons you know about the advantages of piecewise polynomial interpolation over single polynomial interpolation.

Sol. Computationally more efficient and smaller numerical error.