whose endpoints are A and B of a B-spline curve $\boldsymbol{C}(t)$, simply apply the above fine tuning process to the segment between A and B with the fixed endpoints, fixed endpoint tangents and fixed endpoint curvature constraints.

Examples of local fine tuning and their FFD counterparts are shown in Figure 6 and their curvature profiles are drawn in Fingure 6. 6(a) and 6(b) are deformation results by FFD of a bicubic B-spline surface whose grid is shown in light red. 6(c) and 6(d) are results of the fine tuning process of the same cubic B-spline curve (in white). The scalar functions used in these cases, shown at the bottom of the figures, are quadratic B-spline functions. The end-point constraint, as discussed in the previous section, is enforced in both cases.

The results in (a) and (b) show that FFD is clearly better than direct manipulation of the control points in the sense that moving a point of the grid provides smoother shape deformation of the curve than moving a control point. However, the impact of moving a grid point on the curvature profile of a curve is not so clear. Another problem is the selection of the grid size which determines the range of deformation in the FFD process.

The fine tuning technique, on the other hand, does not have these kinds of problems. The region to be fine tuned can be explicitly specified by the user and by dragging a scalar function, the user can adjust the derivative and, consequently, curvature profile of the curve systematically. The deformation process is completely local and unnecessary changes to the curvature profile can be reduced to a minimum, as shown in (c) and (d). Oscillations of the curvature profile could occur when imposing the tangent and curvature constraints at the endpoints of the deformed region. This is because increased constraints reduce the degree of freedom in the fine tuning process. But the oscillations are usually limited to small regions near the endpoints only, as shown in 6(e) and 6(f).

2.3 Repeated Local Fine Tuning

Figures 8(a) and 8(b) show the results of two consecutive local deformations by the fine tuning technique. In 8(a), the left eye of the given quadratic curve (in green) is locally deformed with the tangent boundary constraint to get a new shape (in red). The scalar function used in the deformation process is a linear function. Hence, the new shape of the left eye (in red) is a cubic curve. A degree reduction process [11] is then performed on this new curve to reduce its degree to two again. This new curve, together with a neighboring region, is then locally deformed with the tangent boundary constraint to get the red curve in 8(b).

This example demonstrates that, by performing an automatic degree reduction after each deformation process, one can locally fine tune a curve repeatedly without increasing the degree of the curve. Several techniques are available to evaluate the error induced by the degree reduction process [10]. One can control the number of segments in a fine-tuned curve by inserting a new knot into the curve only when the error of the degree reduction process for a segment exceeds a given tolerance.

2.4 Ornamental Curves

Figure 8(c) shows an example that can not be produced by FFD or simple global scaling. The doubly looped green curve is deformed by gradually decreasing the norm of its tangent for the first half and then gradually increasing the norm of its tangent for the second half to yield the red curve. As the scaling factor at every portion of the curve is strictly controlled by the scalar function, we can generate a geometrically regulated curve that may be used for ornamental purpose.

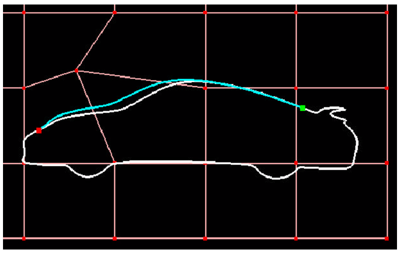
3 Surface Fine Tuning

Basic ideas of the surface fine tuning technique will be discussed in this section. For simplicity, only B-spline surfaces will be considered in this section. But, like the curve case, the technique can be used for all parametric surfaces except a few cases where analytic integration is not possible.

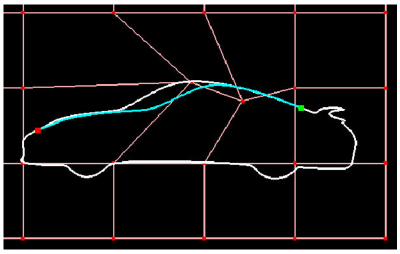
3.1 Curve Based Approach

Skinning is a basic but popular design method. A smooth surface is generated by interpolating (blending) a set of cross-sectional curves with some functional or geometrical continuity constraints. The shape of the resulting surface can be deformed by fine tuning its cross-sectional curves. An example is shown in Figure 6 where three cross-sectional curves drawn in green in (a) are locally fine tuned separately, as shown in (b), to generate the new surface shown in (c). The cross-sectional curves are interpolated with G^1 continuity.

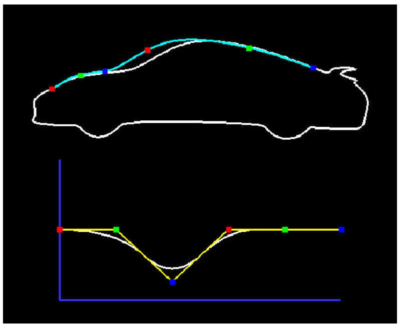
In the skinning process, the cross-sectional curves are usually segmented at the same longitudinal locations to ensure fairness of the resulting surface. In Figure 9, to ensure the same segmentation of the cross-sectional curves



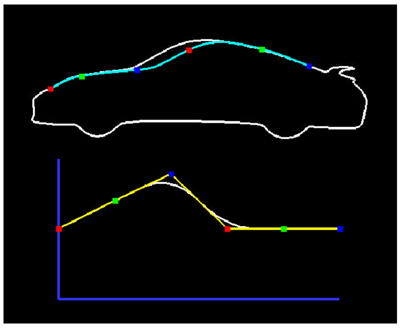
(a) Deformation by FFD (example No.1).



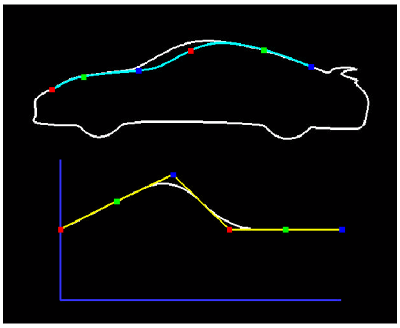
(b) Deformation by FFD (example No.2).



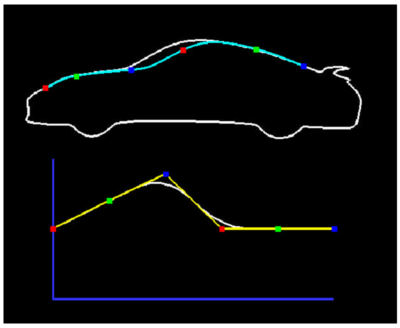
(c) Deformation by Fine Tuning (example No.1.).



(d) Deformation by Fine Tuning (example No.2.).

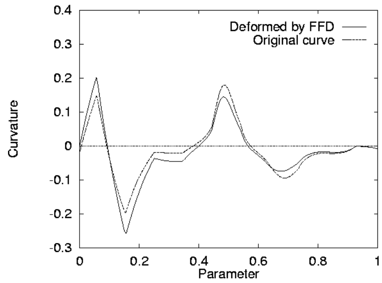


(e) Tangent constraint.

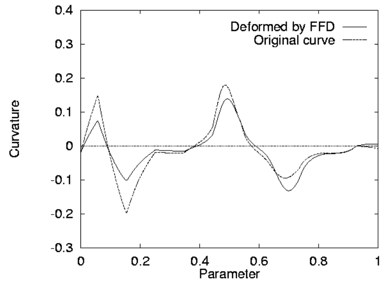


(f) Curvature constraint.

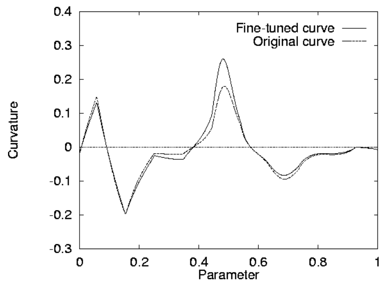
Figure 6. Comparison of fine tuning and FFD. The original curves and deformed curves are in white and cyan, respectively. The endpoints of the deformed curves are fixed. (a) and (b) are deformed by FFD. (c) and (d) are deformed by fine tuning. In (c) and (d), corresponding points of the fine-tuned curve and the control values of the scalar function are displayed in the same colors to show the relationship between the fine-tuned curve and the scalar function. (e) Tangent constraint. (f) Curvature constraint.



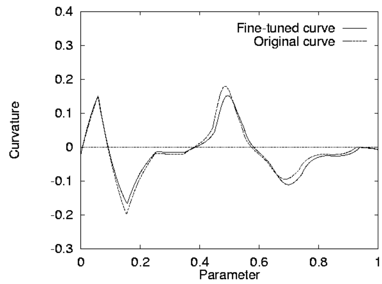
(a) Fig.6(a).



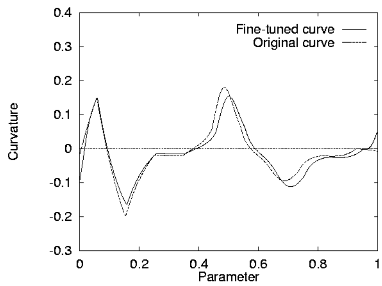
(b) Fig.6(b).



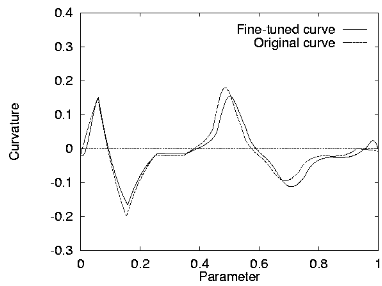
(c) Fig.6(c).



(d) Fig.6(d).



(e) Fig.6(e).

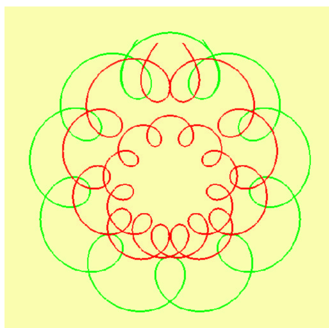


(f) Fig.6(f).

Figure 7. Curvature profiles.

(a) 1st local deformation with the tangent constraint.

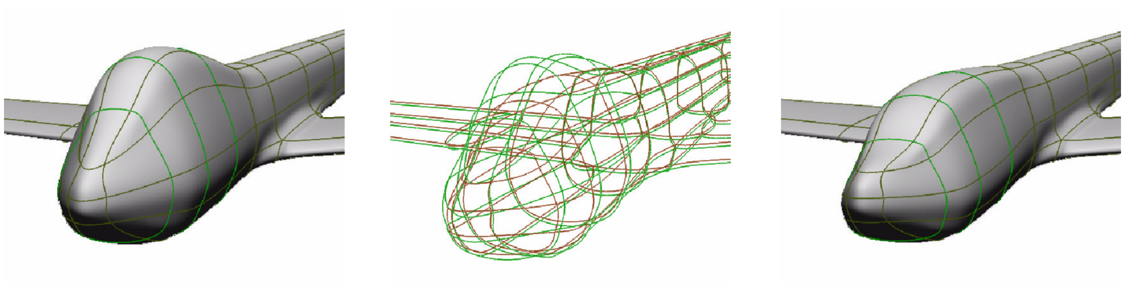
(b) 2nd local deformation with the tangent constraint at different endpoints.



(c) Ornamental curve.

Figure 8. Examples of curve fine tuning. (a) and (b): Repeated local deformation. (c) Ornamental curve.

is kept after the fine tuning process, a scalar function that has the same segmentation as the cross-sectional curves is used in the fine tuning process. A degree reduction process is then performed on the fine-tuned curves to reduce their degrees to the original ones and, consequently, the original segmentation.



(a) Original surface. (b) Comparison of the original and fine-tuned surfaces. (c) Fine-tuned surface.

Figure 9. Surface deformation based on curve fine tuning. In (a), three sectional curves in green are deformed using curve fine tuning technique to get the deformed surface in (c).

3.2 Surface Based Approach

Given a B-spline surface $\boldsymbol{S}(u,v)$, $0 \leq u,v \leq 1$, similar to the curve case, one can control its curvature at any portion by introducing a scalar function $\alpha(u,v) > 0$ as follows:

$$\boldsymbol{T}(u,v) = \boldsymbol{P}_0 + \int_{path} \alpha(u,v)\Big(\frac{\partial \boldsymbol{S}(u,v)}{\partial u} + \frac{\partial \boldsymbol{S}(u,v)}{\partial v}\Big)du dv, \tag{6}$$

where $\boldsymbol{P}_0 = \boldsymbol{S}(0,0)$. Ideally, $\boldsymbol{T}(u,v)$ should be independent of the integration path, i.e., $\partial(\alpha(u,v)\partial\boldsymbol{S}(u,v)/\partial u)/\partial v = \partial(\alpha(u,v)\partial\boldsymbol{S}(u,v)/\partial v)/\partial u$. However, this condition puts a restriction on the variation of $\alpha(u,v)$ and it restricts the user's capability in deforming the surface freely. Hence, instead of Eq.(6), the following definition is adopted for $\boldsymbol{T}(u,v)$:

$$\begin{aligned}\boldsymbol{T}(u,v) &= \boldsymbol{P}_0 + \frac{1}{2}\Big(\int_0^u \alpha(u,0)\frac{\partial \boldsymbol{S}(u,0)}{\partial u}du + \int_0^v \alpha(u,v)\frac{\partial \boldsymbol{S}(u,v)}{\partial v}dv \\ &\qquad + \int_0^v \alpha(0,v)\frac{\partial \boldsymbol{S}(0,v)}{\partial v}dv + \int_0^u \alpha(u,v)\frac{\partial \boldsymbol{S}(u,v)}{\partial u}du\Big) \\ &= \frac{1}{2}(\boldsymbol{T}_{uv}(u,v) + \boldsymbol{T}_{vu}(u,v)),\end{aligned} \tag{7}$$

where

$$\begin{aligned}\boldsymbol{T}_{uv}(u,v) &= \boldsymbol{P}_0 + \int_0^u \alpha(u,0)\frac{\partial \boldsymbol{S}(u,0)}{\partial u}du + \int_0^v \alpha(u,v)\frac{\partial \boldsymbol{S}(u,v)}{\partial v}dv, \\ \boldsymbol{T}_{vu}(u,v) &= \boldsymbol{P}_0 + \int_0^v \alpha(0,v)\frac{\partial \boldsymbol{S}(0,v)}{\partial v}dv + \int_0^u \alpha(u,v)\frac{\partial \boldsymbol{S}(u,v)}{\partial u}du.\end{aligned} \tag{8}$$

Eq.(7) requires two paths to calculate a point $\boldsymbol{T}(u,v)$, therefore, is twice as time-consuming as the one-path approach using either $\boldsymbol{T}_{uv}(u,v)$ or $\boldsymbol{T}_{vu}(u,v)$ only. But the two-path approach provides more flexibility in manipulating the value of the scalar function while the one-path approach might lose fine tuning effect in one parameter direction in some cases. Consider, for example, the case shown in Figure 10 where the value of the scalar function is modified in the region of the parameter space (u,v) painted in blue. In this case, if the one-path approach is adopted, say the red path, one would not get any fine tuning effect in u direction at all.

If fine tuning effect is needed only in a specific direction, one should adopt an adequate one-path integration, instead of the two-path approach.