Figures from 16(g) to 16(l) show another example of fine tuning a Doo-Sabin subdivision surface. Figure 16(l) shows a fine-tuned surface defined by the control mesh in 16(i). The belly and all the wings of the wasp model are deformed. The processing times of the deformations are from a second to 10 seconds, depending on the number of parameters. The example further shows that our fine tuning technique has the power to deform a portion of a complicated object.

To use the fine tuning technique to deform a Catmull-Clark surface, a quadratic scalar function is recommended and the fine-tuned surface should be a subdivision surface based on the biquintic tensor product surface.

## 5 Conclusion

A new deformation-based fine tuning technique for parametric curves and surfaces has been proposed and discussed. The new approach is different from traditional approaches in that the deformation is performed by scaling the derivative of the curve or surface, instead of manipulating its control points. Therefore, the new approach allows direct manipulation of the curvature (and, consequently, fairness) of a curve or surface. The new technique allows fine tuning of a curve or surface without changing the basic shape of its profile and curvature distribution. Precise shaping and deformation, such as making the curvature of a region twice as big, is possible with the new approach. We have shown how to use the new technique to perform fine tuning on an arbitrary portion of a curve or surface. We have also shown how to use the new technique to fine tune subdivision curves and surfaces.

This is the first time a deformation technique based on scaling the derivative of a curve or surface is proposed. To ensure that this is indeed a powerful fine tuning technique, we have tested the new technique on various data sets with various different requirements. In addition to the applications cited above, we also see the possibly of using the new technique for multi-resolution deformation of curves and surface and wavelet analysis. Another possible application area is the fine tuning of implicit curves and surfaces. A difficult task in that area is the selection of differentiation direction for surfaces.

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# Appendix

## (a) Degree elevation

The degree elevation of a B-spline curve is shown here. We follow the notations of Piegl and Tiller [11] in this work.

Let $\boldsymbol{C}_p = \sum_{i=0}^{n} N_{i,p}(u)\boldsymbol{P}_i$ be an end point interpolating (nonperiodic) degree $p$ B-spline curve with respect to the knot vector $U$. To elevate its degree to $p+1$, one needs to construct a knot vector $\hat{U}$ and control points $\boldsymbol{Q}_i$ so that

$$\boldsymbol{C}_p(u) = \boldsymbol{C}_{p+1}(u) = \sum_{i=0}^{\hat{n}} N_{i,p+1}(u)\boldsymbol{Q}_i. \tag{15}$$

Assume $U$ has the following form:

$$U = \{u_0, \cdots, u_m\} = \{a, \cdots, a, u_1, \cdots, u_1, \cdots, u_s, \cdots, u_s, b, \cdots, b\} \tag{16}$$

where the multiplicities of the interior knots are $m_1, \cdots, m_s$, respectively. At a knot of multiplicity $m_i$, $\boldsymbol{C}_p(u)$ is $C^{p-m_i}$ continuous and $C_{p+1}(u)$ must have the same degree of continuity there. Therefore, $\hat{n} = n+s+1$ and

$$\hat{U} = \{u_0, \cdots, u_{\hat{m}}\} = \{a, \cdots, a, u_1, \cdots, u_1, \cdots, u_s, \cdots, u_s, b, \cdots, b\} \tag{17}$$

where $\hat{m} = m+s+2$. The control points $\boldsymbol{Q}_i$ of the degree-elevated curve $C_{p+1}(u)$ are determined by solving the following system of linear equations:

$$\sum_{i=0}^{\hat{n}} N_{i,p+1}(u_j)\boldsymbol{Q}_i = \sum_{i=0}^{n} N_{i,p}(u_j)\boldsymbol{P}_i, \qquad j = 0, \cdots, \hat{n}, \tag{18}$$

where $u_j$ are $\hat{n}+1$ appropriate parameter values. The degree elevation of a NURBS curve can be done similarly and there are more efficient methods to calculate $\boldsymbol{Q}_i$ [11].

In case of a periodic curve, if the knot vector $U$ is $\{u_0, u_1, \cdots, u_m\}$, then $\hat{U}$ is given by

$$\hat{U} = \{u_1, u_1, \cdots, u_{m-1}, u_{m-1}\}. \tag{19}$$

For example, the closed quadratic B-spline curve constructed from the triangular control polygon in Figure 14(a) consists of three segments and its knot vector $U$ is as follows:

$$U = \{-\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}\}. \tag{20}$$

The knot vector $\hat{U}$ and the control points $\boldsymbol{Q}_i$ of the degree elevated cubic B-spline curve are given by

$$U = \{-\frac{1}{3}, -\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, 1, 1, \frac{4}{3}, \frac{4}{3}\}, \tag{21}$$

and

$$\boldsymbol{Q}_{2i} = \frac{5}{6}\boldsymbol{P}_i + \frac{1}{6}\boldsymbol{P}_{i+1}, \boldsymbol{Q}_{2i+1} = \frac{1}{6}\boldsymbol{P}_i + \frac{5}{6}\boldsymbol{P}_{i+1}, \tag{22}$$

for $i = 0, \cdots, 3$. The original quadratic B-spline curve and the degree elevated curve can be regarded as a uniform and a non-uniform B-spline subdivision curve, respectively. The knot intervals of the degree elevated curve are shown in Figure 14(d).

## (b) Approximate conversion from a Doo-Sabin subdivision surface to a non-uniform Catmull-Clark subdivision surface

The approximate conversion from a Doo-Sabin subdivision surface to a non-uniform Catmull-Clark subdivision surface can be performed topologically in the same way as the Doo-Sabin subdivision, but geometrically they are different and different subdivision coefficients are used for the conversion process. We apply to the Doo-Sabin surface a linear B-spline function whose parameter domain is identical to that of the original surface.

For a regular mesh, as the one shown in Figure 17(a), the conversion may be regarded as a degree elevation of a uniform biquadratic surface to a non-uniform bicubic B-spline surface. In the figure, the green lines are patch boundaries of the quadratic B-spline surface. The yellow and red lines are those of the non-uniform cubic B-spline surface whose knot intervals are 1 and 0, respectively. The new control points $\boldsymbol{F}_a$ is given by

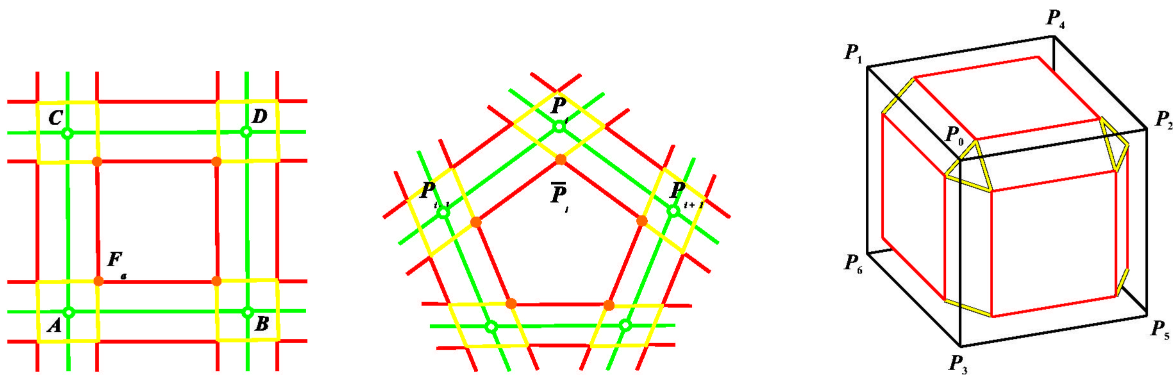
$$\boldsymbol{F}_a = \frac{25}{36}\boldsymbol{A} + \frac{5}{36}(\boldsymbol{B}+\boldsymbol{C}) + \frac{1}{36}\boldsymbol{D} = \frac{(\boldsymbol{V}+2\boldsymbol{A})}{3} + \frac{\boldsymbol{B}+\boldsymbol{C}-\boldsymbol{A}-\boldsymbol{D}}{9}, \tag{23}$$

where $\boldsymbol{V} = (\boldsymbol{A}+\boldsymbol{B}+\boldsymbol{C}+\boldsymbol{D})/4$.

The $n$-sided faces where $n \neq 4$ remain $n$-sided after the conversion in a manner identical to the Doo-Sabin subdivision. In Figure 17(b), the new vertex $\bar{\boldsymbol{P}}_i$ is calculated by

$$\bar{\boldsymbol{P}}_i = (\frac{4}{9}+\frac{1}{n})\boldsymbol{P}_i + \frac{1}{9n}\sum_{j=1, j\neq i}^{n}(5+4\cos(\frac{2\pi|i-j|}{n}))\boldsymbol{P}_j. \tag{24}$$

The yellow and red lines are boundaries of the non-uniform cubic B-spline surface whose knot intervals are 1 and 0, respectively, as the regular mesh case.




(a) Regular mesh. (b) Non-four-sided face. (c) Conversion error.

**Figure 17. The conversion process and its error.**

For a regular mesh, the above conversion is exact in the sense that a Doo-Sabin subdivision surface is exactly converted to a non-uniform Catmull-Clark subdivision surface. However, for an irregular mesh, the conversion is approximate. For example, the point $\boldsymbol{P}_0$ of the Doo-Sabin control mesh in Figure 17(c) will converge to $\boldsymbol{P}_0^\infty$:

$$\boldsymbol{P}_0^\infty = \frac{9}{16}\boldsymbol{P}_0 + \frac{1}{8}(\boldsymbol{P}_1+\boldsymbol{P}_2+\boldsymbol{P}_3) + \frac{1}{48}(\boldsymbol{P}_4+\boldsymbol{P}_5+\boldsymbol{P}_6). \tag{25}$$

The corresponding point $\boldsymbol{Q}_0^\infty$ of the converted Catmull-Clark surface is given by

$$\boldsymbol{Q}_0^\infty = \frac{629}{1152}\boldsymbol{P}_0 + \frac{223}{172}(\boldsymbol{P}_1+\boldsymbol{P}_2+\boldsymbol{P}_3) + \frac{77}{3456}(\boldsymbol{P}_4+\boldsymbol{P}_5+\boldsymbol{P}_6). \tag{26}$$

The difference of these two points is

$$\boldsymbol{P}_0^\infty - \boldsymbol{Q}_0^\infty \approx 0.0165\boldsymbol{P}_0 - 0.00405(\boldsymbol{P}_1+\boldsymbol{P}_2+\boldsymbol{P}_3) - 0.00145(\boldsymbol{P}_4+\boldsymbol{P}_5+\boldsymbol{P}_6). \tag{27}$$

This is small enough for practical CG applications where extremely high degree is not the primary concern. The primary concern is the deformation process.