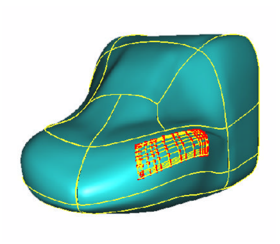
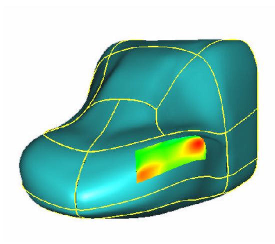
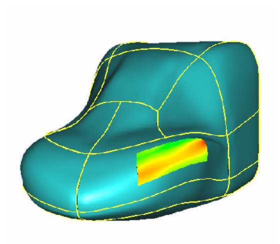
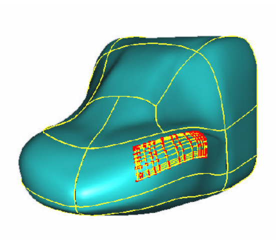
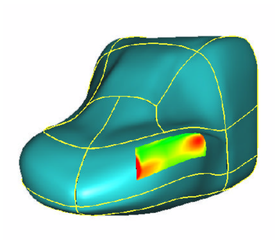
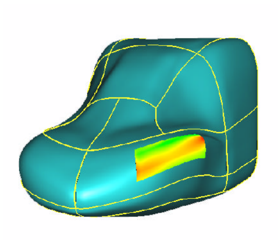

(a) Control points of the fine-tuned surface (boundary constraint).

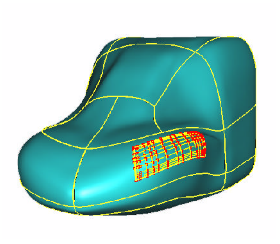

(b) Gaussian curvature distribution (boundary constraint).

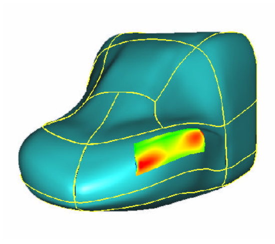

(c) Mean curvature distribution (boundary constraint).


(d) Control points of the fine-tuned surface (tangent constraint).


(e) Gaussian curvature distribution (tangent constraint).


(f) Mean curvature distribution (tangent constraint).


(g) Control points of the fine-tuned surface (curvature constraint).


(h) Gaussian curvature distribution (curvature constraint).

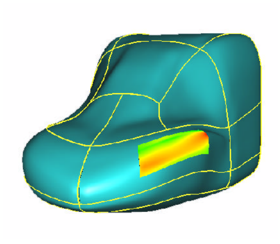

(i) Mean curvature distribution (curvature constraint).

Figure 13. Examples of local fine tuning. (a), (b) and (c) are the control points, Gaussian and mean curvature distribution of the fine-tuned surface with the boundary constraint. (d), (e) and (f), (g), (h) and (i) are those with the tangent and curvature constraints, respectively.

the symbolic operators for NURBS curves and surfaces proposed by Piegl and Tiller [12] for typical B-spline combinations.

Curve	deg. (orig.)	#ctl. pnt.	deg. (sca.)	#ctl. val.	time(second)
Face (Fig.2)	2	181	1	2	0.0052
Car (Fig.6)	3	72	2	6	0.013
Double-loop (Fig.8(c))	3	159	2	3	0.014

Surface	deg. u, v (orig.)	#ctl. pnt.	deg. u, v (sca.)	#ctl. val.	time(second)
Fig.11	3, 3	$4 \times 4 = 16$	2, 2	$3 \times 3 = 9$	0.16
	3, 3	$4 \times 4 = 16$	2, 2	$3 \times 4 = 12$	0.33
	3, 3	$5 \times 5 = 25$	2, 2	$3 \times 3 = 9$	0.49
Train (Fig.13)	3, 3	$4 \times 8 = 32$	2, 2	$3 \times 3 = 9$	0.61
Train (Fig.12)	3, 3	$6 \times 12 = 72$	2, 2	$3 \times 4 = 12$	2.74

Table 1. Table of degrees, numbers of control points of the original shape, degrees and numbers of control values, and processing times of fine tuning process (differentiation, product and integration).

4 Fine Tuning Subdivision Curves and Surfaces

Subdivision surfaces are powerful tools for graphical modeling and animation because of their scalability, numerical stability, simplicity in coding and, especially, their ability to represent complex shape of arbitrary topology [3]. A surface manipulation technique that does not work for subdivision surfaces would not be complete. In this section, we will show that the new fine tuning technique works for subdivision curves and surfaces as well. Three issues have to be addressed when dealing with closed subdivision curves and surfaces, namely, (1) how should the domain of the scalar function α be specified, (2) how should the product of the original shape and the scalar function be performed, and (3) how to keep the fine-tuned curve or surface closed. We will address these issues while we do the illustration.

4.1 Fine Tuning Subdivision Curves

We will use Chaikin's algorithm as an example to illustrate the fine tuning process of a subdivision curve. Chaikin's algorithm [2] generates a quadratic B-spline curve from a control polygon through recursive subdivision. Each subdivision step generates two new points on each polygon leg. If there are $n+1$ vertices $\boldsymbol{p}_i^j$, $i = 0, ..., n$, after the j-th recursive subdivision, then the two new points generated for the polygon leg $\boldsymbol{p}_i^j \boldsymbol{p}_{i+1}^j$ are defined as follows:

$$\boldsymbol{p}_{2i}^{j+1} := \frac{3}{4}\boldsymbol{p}_i^j + \frac{1}{4}\boldsymbol{p}_{i+1}^j, \tag{9}$$

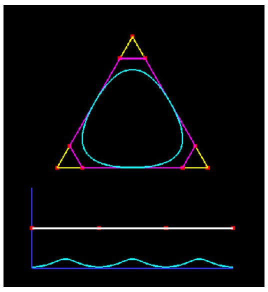
$$\boldsymbol{p}_{2i+1}^{j+1} := \frac{1}{4}\boldsymbol{p}_i^j + \frac{3}{4}\boldsymbol{p}_{i+1}^j. \tag{10}$$

The subdivision curve generated by Chaikin's algorithm is a quadratic B-spline curve. Hence, if a linear B-spline scalar function is used in the fine tuning process of the subdivision curve, the resulting curve would be a cubic B-spline curve. Consider, for example, the triangular control polygon drawn in yellow in Figure 14(a). The subdivision curve generated from this control polygon has three segments. If we apply as a scalar function a linear B-spline function of three segments whose knot vector is identical to that of the original curve but without the first and the last knots, we will obtain a three-segment, non-uniform cubic B-spline curve whose knots are all of multiplicity two (see Appendix (a)). Here the domain issue of the scalar function is resolved by adopting the same domain as the original curve, and the product issue is resolved by treating it as a degree elevation process which converts a uniform quadratic B-spline curve to a non-uniform cubic B-spline curve.

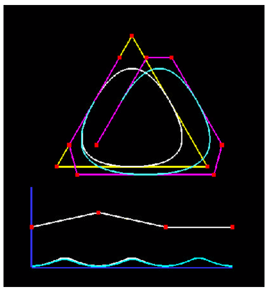
The conversion of the control polygon of a quadratic B-spline curve to that of a non-uniform cubic B-spline curve is accomplished by generating two new points defined as follows for each polygon leg $\boldsymbol{p}_i \boldsymbol{p}_{i+1}$,

$$\boldsymbol{q}_{2i} := \frac{5}{6}\boldsymbol{p}_i + \frac{1}{6}\boldsymbol{p}_{i+1}, \tag{11}$$

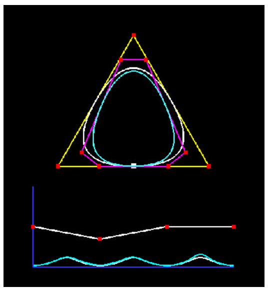
$$\boldsymbol{q}_{2i+1} := \frac{1}{6}\boldsymbol{p}_i + \frac{5}{6}\boldsymbol{p}_{i+1}, \tag{12}$$



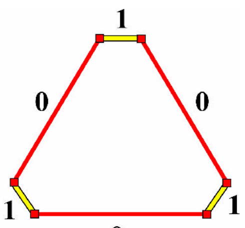
(a) Original control polygons (Quadratic: yellow; non-uniform cubic: magenta).



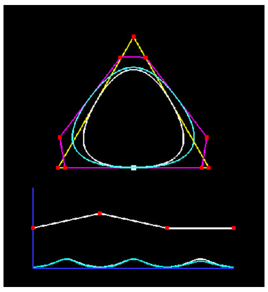
(b) Deformed subdivision curve (the start point is fixed).



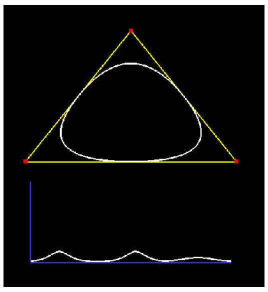
(c) Deformed curve based on the spring model.



(d) Knot intervals for the non-uniform cubic B-spline subdivision curve.



(e) Deformed curve using the same setting as (b) but based on the spring model.



(f) Deformed curve by changing the length of the bottom leg of the original curve.

Figure 14. Examples of subdivision curve fine tuning.

in a manner similar to Chaikin's algorithm. The converted control polygon is the control polygon of a non-uniform cubic B-spline subdivision curve proposed by Sederberg et al. [16]. For a periodic cubic B-spline curve, there is an one-to-one correspondence between the edges of the control polygon and the segments of the curve. Following the same notation of [16], a knot interval between two consecutive knots is assigned to each control edge of the converted curve, as shown in 14(d).

Note that for a periodic quadratic B-spline curve, there is an one-to-one correspondence between the vertices of the control polygon and the segments of the curve. However, for a periodic linear B-spline curve the correspondence is between the edges of the control polygon and the segments. To define a correspondence between the scalar function and the subdivision curve, we assume there is a virtual vertex between each two consecutive segments of the subdivision curve and assign a control value of the scalar function to the virtual vertex instead of the vertex of the original curve. Using the control values assigned at the ends of a quadratic segment of the original curve, one can define a linear function by interpolating these control values and use the value of this linear function to scale the derivative of corresponding point of the segment. This way, one can fine tune each segment of the curve and, consequently, the entire curve.

Figure 14(a) shows the original curve and its invariant version fine tuned by a linear identity scalar function. The graph beneath it is the scalar function with its control values (points) shown in red. The first and last control values are identical to ensure that the scalar function is also periodic. 14(b) shows fine-tuned curves generated by increasing the control value corresponding to the virtual vertex between the two segments joined at the bottom of the original curve. The curve is no longer closed after the fine tuning process. The method described in Section 2 for the end-point constraint does not work in this case. A different approach has to be used.

4.2 Closed Curves

The closedness issue of a fine tuned subdivision curve is discussed in this subsection. To ensure that a closed subdivision curve remains closed after the fine tuning process, one needs the capability of restructuring the control polygon of the curve geometrically (but not topologically) without inducing large change on the curvature profile of the resulting, fine-tuned curve. This can be achieved by assuming that legs of the control polygon are made of flexible springs and adjacent legs are joined with rotational springs. The control points of the fine-tuned curve are